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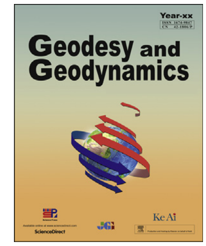

Geopotential difference determination using optic-atomic clocks via coaxial cable time transfer technique and a synthetic test

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ABSTRACT

According to the general theory of relativity, two clocks placed at two different positions with different geopotentials run at different rates. Thus one can determine the geopotential difference between these two points by comparing the running rates of the two clocks. Using the most precise optic-atomic clocks whose stability achieves 10^{-18} level and the time transfer technique with comparison accuracy higher than 10ps level by 100 m coaxial cable, the relativistic geodesy method for determining the geopotential may be realizable in the near future. In this paper, we propose, design and describe in detail an approach for determining the geopotential difference between two points based upon a simulation experiment. We select two stations A and B whose ground distance is within 100 m, height difference being about 30 m. Each station is equipped with an atomic clock whose instability is $3.2 \times 10^{-16}/\sqrt{\tau}$ (where τ is time in second). And the two stations are connected with a coaxial cable for time transfer. Our simulation experiment results show that the accuracy could reach 0.16 m^2/s^2 (equivalent to 1.6 cm in height) level after a four-hour period of observation.

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1. Introduction

The Earth's geopotential (gravity potential) field is the foundation of the definition of the geoid and world height system. Based on the data of TOPEX/Poseidon satellites, the geopotential constant value W_0 (geopotential constant on the geoid) is determined with an uncertainty of $\pm 0.5 \mathrm{m}^2\mathrm{s}^{-2}$ [1]. Conventional method – leveling combining with gravimetry – for determining the geopotential or orthometric height is laborious [2]. To investigate alternative approaches for measuring geopotential, Bjerhammar [3] put forward an idea that the geopotential can be determined by precise clocks based on the general relativity theory (GRT). Later, Shen and his colleagues [4,5] suggested that the geopotential could be determined by gravity frequency shift. The relativistic approaches have advantages in some extend compared to the conventional leveling method, especially for world height datum unification. However, for a long time the limited stability of clocks constrains the realization of relativistic approach. In order to determine geopotential with the accuracy of 0.1 $\mathrm{m}^2\mathrm{s}^{-2}$ (equivalent to 1 cm in height), we need not only time transfer technique with high precision, but also time-keeping system (atomic clocks) with a stability better than 10^{-18}.

Fortunately, the highly developed atomic clock technology and time transfer technique make it prospective in the near future to actually realize the determination of the potential using the relativistic geodetic approach. To date, the uncertainty of optical-atomic clocks achieves around 1.6×10^{-18} [6–9], and the time transfer techniques with comparison accuracy better than 10ps are available [10–12]. These precise clocks and advanced signal-transmitting techniques are potential for determining geopotential difference between two stations (or points) with high accuracy via a relativistic geodetic approach.

In this paper, we propose an approach for determining the geopotential difference between two points. In Section 2, after explaining the fundamental of our approach, we describe in detail the set up of experiment and the method in determining the geopotential difference. In Section 3, we discuss and analyze all possible errors in determining the geopotential difference using the proposed approach. Then in Section 4, a simulation experiment is provided based upon the method, and the accuracy is estimated. In the last section, we summarize the main results and discuss some problems that need further investigations.

2. Fundamental

According to GRT, the running rate of a precise clock is related to the geopotential where it is located. Suppose there are two stations A and B with the geopotentials W_A and W_B respectively. Assume stations A and B are equipped with precise time-keeping systems, simply clocks C_A and C_B respectively, which record time T_A and time T_B respectively. Then the following equation holds [3,13]:

$$\frac{dT_A/dt}{dT_B/dt} = \frac{1 + c^{-2}W_A}{1 + c^{-2}W_B} \tag{1}$$

where dT_A/dt and dT_B/dt denote the time elapsing speeds of clocks C_A and C_B, c is the speed of light in vacuum. Based on equation (1), if we choose the geopotential W_A at station A as datum, the geopotential W_B at station B can be expressed as follows:

$$W_B = c^2\left(\frac{dT_B}{dT_A} - 1\right) + \frac{dT_B}{dT_A}W_A \tag{2}$$

Then the geopotential difference ΔW_{AB} between A and B can be obtained:

$$\Delta W_{AB} = W_B - W_A = \left(c^2 + W_A\right)\left(\frac{dT_B}{dT_A} - 1\right) \approx c^2\left(\frac{dT_B}{dT_A} - 1\right) \tag{3}$$

To simplify equation (3), we take a substitution $a = \frac{dT_B}{dT_A} - 1$, then, we have:

$$\Delta W_{AB} = ac^2 \tag{4}$$

From equation (4), it is clear that the determination of ΔW_{AB} requires a precise measurement of a, which represents the difference between the time elapsing speeds of clocks C_A and C_B located respectively at stations A and B. In order to measure the value of a, we may set the time of clock C_A as standard, and define a standard time duration ΔT. When the time of clock C_A elapses a standard time duration $\Delta T_A = \Delta T$, the time of clock C_B elapses $\Delta T_B = \Delta T'$. Then if the time duration difference $\Delta T' - \Delta T$ is precisely measured, the parameter a can be determined by the following equation:

$$a = \frac{\Delta T_B - \Delta T_A}{\Delta T_A} = \frac{\Delta T' - \Delta T}{\Delta T} \tag{5}$$

where the quantity of ΔT is a priori given.

However, there are two key problems to realize this measurement. First, the clocks need to be in high stability for keeping the standard time duration ΔT stable, for effective measurement of a small value. Second, we need a reliable time transfer technique to guarantee that when clock C_A records a time duration ΔT, clock C_B could record a corresponding time duration $\Delta T'$ that obeys GRT in high accuracy.

In order to solve these two problems, first we apply the most recently developed optic-atomic clocks as time keeping systems, whose stability is reported as in 10^{-18} level [8,9]. Second we employ a time transfer technique proposed by [11] that time signal is transferred by a coaxial cable in the forms of pulse current. This technique is relatively economical and easier to be realized compared to other time transfer technique such as optical fiber transfer technique [14–16] or laser transfer technique [12,17]. And its accuracy for short distance (100 m) is satisfactory, reaching 10ps in 1 s [11].

Here we need two a priori synchronized atomic clocks placed at stations A and B respectively. We apply a two-way time transfer technique for comparing these two clocks. The basic idea of time transfer method is shown as Fig. 1 (modified from [11]).

In Fig. 1, τ_A and τ_B denote the propagation delays of signals or electric current pulse (simply pulse) in each station, τ_C denotes the propagation delay of pulse going through the connection cable. If we only consider the time delay τ_C and

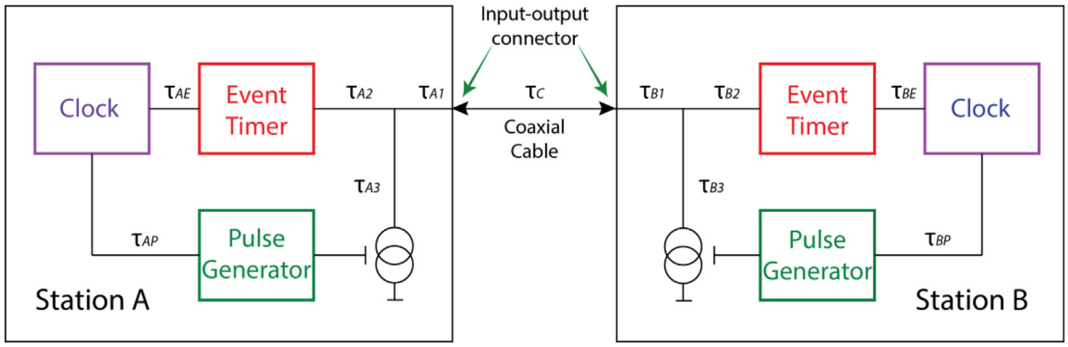

Fig. 1 – Schematic figure of time transfer (modified from [11]). Each station includes an event timer which measures the arrival time of pulses when the pulses come to the input of the event timer, and includes a pulse generator which can drive a transmission line through a current driver. Event timer and pulse generator at any station are connected with an atomic clock. Stations A and B are connected by coaxial cable.

ignore all other delays and error sources, then assume that each of the pulse generators A and B emits a pulse signal at almost the same time, the timer A respectively receives two signals from pulse generators A and B at time TAa_0 and time TAb_0 recorded by clock C_A, the timer B respectively receives two signals at time TBa_0 and time TBb_0 recorded by clock C_B. Set the reading difference of clock C_A and C_B as Δ_0 for the first comparison, then the following equations hold:

$$TAa_0 + \tau_C = TBa_0 + \Delta_0 \tag{6}$$

$$TBb_0 + \Delta_0 + \tau_C = TAb_0 \tag{7}$$

Based on equations (6) and (7) the propagation delay τ_C is cancelled and the reading difference Δ_0 is obtained:

$$\Delta_0 = \frac{1}{2}(TAa_0 + TAb_0 - TBa_0 - TBb_0) \tag{8}$$

After a precise time interval ΔT from the first emission, the pulse generators A and B emit second pulse signals. Similarly, the timer A respectively receives two signals at time TAa_1 and time TAb_1 recorded by clock C_A, whereas the timer B respectively receives two signals at time TBa_1 and time TBb_1 recorded by clock C_B. Then the difference of second time comparison reads:

$$\Delta_1 = \frac{1}{2}(TAa_1 + TAb_1 - TBa_1 - TBb_1) \tag{9}$$

Obviously the difference of the two measurements $\Delta_1 - \Delta_0$ reflects the difference of time elapsing speed in a standard time duration ΔT. Consider the definition of parameter a from equation (5), we have:

$$a = \frac{\Delta_1 - \Delta_0}{\Delta T} \tag{10}$$

It is noted that equation (10) does not hold strictly, due to the fact that the time interval ΔT controlled by clocks C_A and C_B is not in the same length strictly. However, since the difference is relatively small compared to ΔT itself, and ΔT is a denominator in equation (10), this kind of error can be safely neglected.

If we continue the measurements described above and obtain a set of time comparison results Δ_0, Δ_1, $\Delta_2 \ldots \Delta_n$, then the random errors of the measurements can be effectively constrained by least squares estimation.

3. Error analysis

In the sequel, we analyze the time transfer process between A and B as described in Fig. 1

The delay of signal from the pulse generator A to the input–output connector A is $\tau_{A1}+\tau_{A3}$, and the delay of signal from the pulse generator A to the input of event timers is $\tau_{A2}+\tau_{A3}$. Similarly the delay of signal from the pulse generator B to the input–output connector B is $\tau_{B1}+\tau_{B3}$, and the delay of signal from the pulse generator B to the input of event timers B is $\tau_{B2}+\tau_{B3}$. The line connecting clock A and pulse generator A has delay τ_{AP}, and the line connecting clock A and time event A has delay τ_{AE}. Similarly the line connecting clock B and pulse generator B has delay $\tau_{B2}+\tau_{B3}$, and the line connecting clock B and time event B has delay τ_{BE}. These delays are relatively small and stable if we effectively keep the environments in the two stations stable. And we can make various experiments to measure the physical properties of these lines in advance in order to estimate their values. The input–output connectors of stations A and B are connected by a coaxial cable with unknown delay τ_C, which is relatively large and unstable. Fortunately τ_C could be cancelled out by two-way approach and have no influence on the clocks comparison as analyzed in the sequel.

Besides the delays of τ_A, τ_B and τ_C we should also consider delays at the inputs of the event timers caused by finite slew rates of the incoming pulses [10]:

$$\tau_{Aa} = \frac{C_A}{S_{Aa}}, \quad \tau_{Ba} = \frac{C_B}{S_{Bb}}, \quad \tau_{Ab} = \frac{C_A}{S_{Ab}}, \quad \tau_{Ba} = \frac{C_B}{S_{Ba}} \tag{11}$$

where C_A and C_B are trigger thresholds of the timers A and B respectively, S_{Aa} and S_{Bb} are the slew rates of pulses generated by the pulse generators A and B observed at the inputs of the timers A and B, respectively, S_{Ab} and S_{Ba} are the slew rates of pulses generated by pulse generators B and A observed at the inputs of the timers A and B, respectively. As the slew rate could be degraded when a pulse goes through a long cable, these additional delays cannot be neglected.

We assume that a time transfer process starts at time t_{ai} recorded by clock C_A and at time t_{bi} recorded by clock C_B, then the pulse generators A and B will initiate pulses at time $t_{ai}-\tau_{AP}$ and time $t_{bi}-\tau_{BP}$, respectively. The timer A may detect two pulse signals from pulse generators A and B at the time TAa_i and time TAb_i recorded by clock C_A, namely:

$$TAa_i = t_{ai} + \tau_{AP} + \tau_{A3} + \tau_{A2} + \tau_{Aa} + \tau_{AE} \tag{12}$$

$$TAb_i = t_{bi} + \tau_{BP} + \tau_{B3} + \tau_{B1} + \tau_C + \tau_{A1} + \tau_{A2} + \tau_{Ab} + \tau_{AE} + \Delta_i \tag{13}$$

where Δ_i denotes the time difference between clocks C_A and C_B at the comparison epoch. Similarly event timer B will detect two pulse signals from pulse generators A and B at the time TBa_i and time TBb_i recorded by clock B:

$$TBa_i = t_{ai} + \tau_{AP} + \tau_{A3} + \tau_{A1} + \tau_C + \tau_{B1} + \tau_{B2} + \tau_{Ba} + \tau_{BE} - \Delta_i \tag{14}$$

$$TBb_i = t_{bi} + \tau_{BP} + \tau_{B3} + \tau_{B2} + \tau_{Bb} + \tau_{BE} \tag{15}$$

Based on equations (12)–(15), the time t_{ai}, time t_{bi} and the delays, τ_{AP}, τ_{BP}, τ_{A3}, τ_{B3}, τ_C, are cancelled out, therefore the time difference Δ_i can be expressed as:

$$\Delta_i = \frac{1}{2}(TAa_i + TAb_i - TBa_i - TBb_i) + (\tau_{B2} - \tau_{A2}) + (\tau_{BE} - \tau_{AE}) + \frac{1}{2}(\tau_{Bb} + \tau_{Ba}) - \frac{1}{2}(\tau_{Ab} + \tau_{Aa}) \tag{16}$$

Assume $\tau_{A2}=\tau_{B2}$, $\tau_{AE}=\tau_{BE}$, $\tau_{Ab}+\tau_{Aa}=\tau_{Bb}+\tau_{Ba}$, then we get the estimation value $\widehat{\Delta}_i$ of the clock record difference as

$$\widehat{\Delta}_i = \frac{1}{2}(TAa_i + TAb_i - TBa_i - TBb_i) \tag{17}$$

thus the error e_{Ti} introduced from the time transfer system can be written as

$$e_{Ti} = \Delta_i - \widehat{\Delta}_i = (\tau_{B2} - \tau_{A2}) + (\tau_{BE} - \tau_{AE}) + \frac{1}{2}(\tau_{Bb} + \tau_{Ba}) - \frac{1}{2}(\tau_{Ab} + \tau_{Aa}) \tag{18}$$

In order to minimize the error e_{Ti}, the experimental equipment in both stations should be identical, and the temperature of the two stations should be kept same and stable. And e_{Ti} can be further decreased by multiple measurements to make average. In a practical operation reported by Pánek and his colleagues [10], the error e_{Ti} fluctuates around 7.4ps with the standard deviation of 2.9ps in a length of 100 m for duration of 1 s.

Besides the time transfer error e_{Ti}, another important error source is the clock error e_{Ci}. The instability of the atomic clock we apply is $3.2 \times 10^{-16}/\sqrt{\tau}$ (for τ in second) [7]. Finally, the error introduced from time measurement can be limited to 4fs [18], which is safely neglected.

4. Experiment setup and simulating experiment

The geopotential difference determination experiment can be conducted between almost any two stations with certain height difference within around 100 m in length. Two stations should be set at the two stations with the setup described in Fig. 1, and be connected with high performance coaxial cable [19]. Those temperature dependence of a length of 100 m is within ±5 ps/K [11]. At the very beginning of the experiment, the atomic clocks at the two stations should be synchronized, that means the two clocks should run at the same speed when they are placed at the same station. This step should be carried out carefully and the precision of the synchronization will directly influence the precision of measurement results. There might be many different methods to synchronize the two clocks. One possible and economical method is placing the two clocks together at station A first, then transport one clock to station B after synchronization. To estimate the synchronization error of this transportation process, we can then transport the clock in station B back to station A for a second comparison. We can repeat this transportation process several times to reduce random errors. After the pre-synchronization process is done, we can start the experiment using the approach as described in Section 2. It is noted that in our simulating experiment we didn't consider the synchronization error, for there are no data for estimating the precision in clock transportation process currently. It requires further experiment to investigate the influence caused by the a priori synchronization errors.

In order to estimate the accuracy in determining the geopotential difference between A and B using the approach proposed in this study, we made several simulating experiments. Assume the true value of geopotential difference between stations A and B is ΔW_{AB}=300 m^2s^{-2} (about 30 m in height). Then the true value of a is 3.33×10^{-15} (based on equation (4)). In the simulating observation, we made (n+1) observations and obtained a set of simulation data $\widehat{\Delta}_0$, $\widehat{\Delta}_1$... $\widehat{\Delta}_n$, which are calculated from equation (10) with random noises to simulate the influences of time transfer errors e_{T0}, e_{T1}... e_{Tn} and clock errors e_{C0}, e_{C1}... e_{Cn} as estimated values. Based on the reports from previous studies [7,10], we assume e_T~N(7.4, 2.9) ps, $e_C \sim N(0,\ 3.2\times10^{-16}/\sqrt{\tau})$. Noting that although we set the standard time duration ΔT (say 1s), the clock instability the ΔT is not a stable and precise value in each measurement. Thus the time interval we use is in fact an estimated value $\Delta\widehat{T}$, which differs from the true values ΔT_0, ΔT_1... ΔT_n for each corresponding measurement. Then according to equation (10), the simulation observation data $\widehat{\Delta}_0$, $\widehat{\Delta}_1$... $\widehat{\Delta}_n$ are calculated by the following procedures

$$\begin{aligned} \widehat{\Delta}_0 &= \Delta_0 + e_{T0} + e_{C0,\tau=1} \\ \widehat{\Delta}_i &= \Delta_0 + a(i + e_{Ci,\tau=i}) + e_{Ti} + e_{Ci,\tau=1}, \quad (i = 1, 2...n) \end{aligned} \tag{19}$$

where the initial value Δ_0 is given arbitrarily. After generating the simulated data (as "observations"), we estimate the value of $\widehat{a}$ from the "observations" $\widehat{\Delta}_0$, $\widehat{\Delta}_1$... $\widehat{\Delta}_n$. According to equation (10), we get the estimated value $\widehat{a}_i$:

$$\widehat{a}_i = \frac{\widehat{\Delta}_i - \widehat{\Delta}_{i-1}}{\Delta\widehat{T}_i}, \quad (i = 1, 2...n) \tag{20}$$

In order to obtain an optimal estimation of $\widehat{a}$, we use least squares estimation. Equation (20) can be modified and expressed as follows:

$$\widehat{\Delta}_i - \widehat{\Delta}_0 = \widehat{a}i + e_i, \quad (i = 0, 1...n) \tag{21}$$

where e_i is the error term in the observation equation. Based on equation (21), the optimal solutions can be obtained by requiring that the estimated values $\widehat{a}$ and $\widehat{b}$ should minimize the quantity

$$\sum_{i=1}^{n} v_i^2 = \sum_{i=1}^{n} \left(\widehat{\Delta}_i - \widehat{\Delta}_0 - \widehat{a}i - \widehat{b}\right)^2 \tag{22}$$

So, set the partial derivative of equation (22) with respect to $\widehat{a}$ and $\widehat{b}$ as zero:

$$\begin{cases} \dfrac{\partial \sum_{i=1}^{n} v_i^2}{\partial \widehat{a}} = -2\sum_{i=1}^{n}\left(\widehat{\Delta}_i - \widehat{\Delta}_0 - \widehat{a}i - \widehat{b}\right)\cdot i = 0 \\ \dfrac{\partial \sum_{i=1}^{n} v_i^2}{\partial \widehat{b}} = -2\sum_{i=1}^{n}\left(\widehat{\Delta}_i - \widehat{\Delta}_0 - \widehat{a}i - \widehat{b}\right) = 0 \end{cases} \tag{23}$$

Equation (23) can be simplified as follows:

$$\begin{cases} n\widehat{b} + \widehat{a}\sum_{i=1}^{n} i = \sum_{i=1}^{n}(\widehat{\Delta}_i - \widehat{\Delta}_0) \\ \widehat{b}\sum_{i=1}^{n} i + \widehat{a}\sum_{i=1}^{n} i^2 = \sum_{i=1}^{n}(\widehat{\Delta}_i - \widehat{\Delta}_0)\cdot i \end{cases} \tag{24}$$

Solve equation (24), we have

$$\begin{aligned} \widehat{a} &= \frac{S_{xy}}{S_{xx}} \\ \widehat{b} &= \frac{1}{n}\sum_{i=1}^{n}(\widehat{\Delta}_i - \widehat{\Delta}_0) - \frac{1}{2}\widehat{a}(n+1) \end{aligned} \tag{25}$$

where

$$\begin{aligned} S_{xy} &= \sum_{i=1}^{n}(\widehat{\Delta}_i - \widehat{\Delta}_0)\cdot i - \frac{(n+1)}{2}\sum_{i=1}^{n}(\widehat{\Delta}_i - \widehat{\Delta}_0) \\ S_{xx} &= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)^2}{4} \end{aligned} \tag{26}$$

The corresponding residual standard deviation is estimated as:

$$\sigma_s = \frac{1}{\sqrt{n-2}}\sqrt{\sum_{i=1}^{n}\left(\widehat{\Delta}_i - \widehat{\Delta}_0 - \widehat{a}i - \widehat{b}\right)^2} \tag{27}$$

And the standard deviation of $\widehat{a}$ is:

$$\sigma_{\widehat{a}} = \frac{\sigma_s}{\sqrt{S_{xx}}} \tag{28}$$

Then, according to equation (4), we have the estimated value $\Delta\widehat{W}_{AB}$:

$$\Delta\widehat{W}_{AB} = \widehat{a}c^2 \tag{29}$$

and the corresponding accuracy reads

$$\sigma_{\Delta\widehat{W}_{AB}} = \sigma_{\widehat{a}}\cdot c^2 \tag{30}$$

The difference between the estimated value $\Delta\widehat{W}_{AB}$ and the true value ΔW_{AB} reads

$$DW = \Delta W_{AB} - \Delta\widehat{W}_{AB} \tag{31}$$

The simulating experiment described above is carried out in Matlab software. From the initial time to a time duration, we made calculations every minute to see how the accuracy is improved with the increasing times of comparison. The results are illustrated in Fig. 2.

Due to errors contained in the simulated observation data $\widehat{\Delta}_t$ and relatively few data for estimation, the results of the first hour fluctuates with large DW values that are not satisfactory. However, as time goes by we have more simulated observation data for calculation, the random error can be efficiently reduced by least squares estimate. As a result, DW becomes smaller and smaller, achieving stable and close to zero, which gives values of $\Delta\widehat{W}_{AB}$ with high accuracy. Thus in Fig. 2, we omit the results of the first hour and illustrate the results from after the first hour to the 8th hour. In an observation of 4 h, the value of DW is about 0.2m^2s^{-2} (equivalent to 2 cm in height). And when the observation time lasts for 8 h, DW decreases to about 0.02 m^2s^{-2} (equivalent to 2 mm in height).

To estimate the stability of our method, we made the simulating experiment ten times and take the observation time as 4 h for each experiment. The results are shown in Fig. 3.

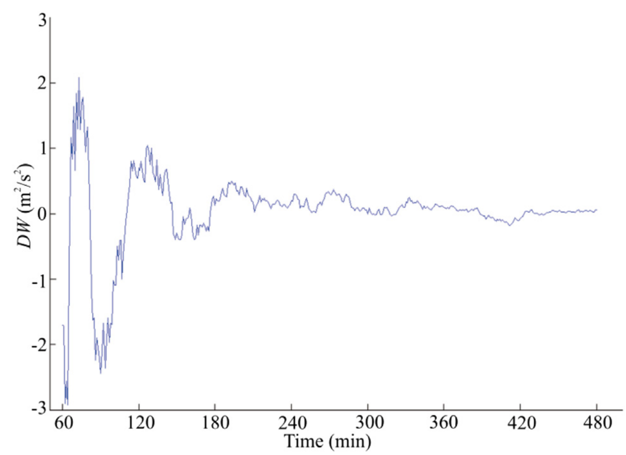

Fig. 2 – Results of a simulating experiment proceeds for 8 h. The absolute value of DW (which indicates accuracy) decreases as time goes by, because there are more virtual observation data $\widehat{\Delta}_t$ for estimation.

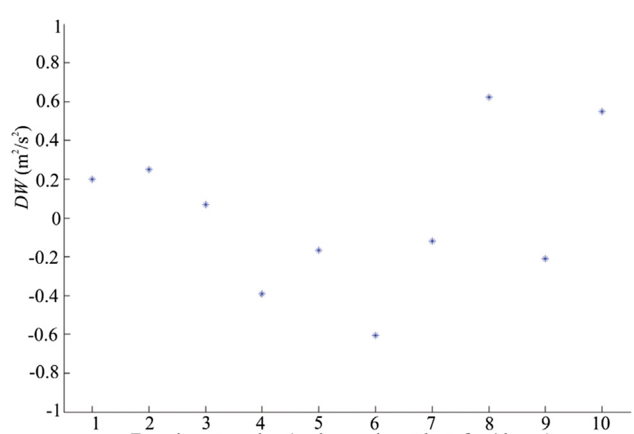

Fig. 3 – Results of ten experiments, which fluctuate around zero with the mean value of 0.0196 m^2s^{-2}, and the standard deviation of 0.1555 m^2s^{-2}. Each experiment lasts for 4 h.

Table 1 – Results of ten experiments.

Times of exp.	Max	Min	Mean	RMS
10	0.6228 m^2s^{-2}	−0.6017 m^2s^{-2}	0.0196 m^2s^{-2}	0.1555 m^2s^{-2}

The experiment results illustrated in Fig. 3 show a satisfactory stability, which are listed in Table 1. The mean value of the ten experiments is 0.0196 m^2s^{-2}, with the RMS (root mean square) of 0.1555 m^2s^{-2} (equivalent to 1.6 cm in height). The minimum and maximum deviations (in the 6th and 8th experiments) are about 0.6 m^2s^{-2} (equivalent to 6 cm in height).

5. Conclusion

In this paper we proposed a relativistic approach to determine the geopotential difference between two stations in short distance via coaxial cable time transfer technique. The result in our simulation experiment is satisfactory, reaching 2 cm in precision after four-hours "observations". However, in real practice we need to consider more environment factors that might influence the precision of the measurement. First, a fundamental requirement in this approach is that two atomic clocks placed at different stations should be a priori synchronized. The uncertainty in the synchronization will influence the precision of final result. Although we have proposed a method to synchronize two atomic clocks in Section 1, the precision of this method remains unknown and need further exploration. Second, in our simulation experiment we simplified the uncertainty of the atomic clocks and assume that their random patterns satisfy Gaussian normal distribution. In fact the uncertainty of atomic clocks is more complicated [20,21] and might cause larger unexpected errors in results.

Nevertheless the relativistic approach is very promising and worth more experiment for development. The clock technology is developing in rapid speed; atomic clocks might be more precise and portable in the near future. The time and frequency transfer techniques are also developing rapidly. In this paper we only proposed one possible method to transfer time signals. In practical application the time signals can be transferred through optical fiber [22], any medium or even free space, provided the precision meets requirement. A very promising mission is to transfer time signals among stations distributed worldwide on ground of the Earth via satellites as signal transmitting conveyors.

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